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Generating asymptotically plane wave spacetimes

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Abstract: In an attempt to study asymptotically plane wave spacetimes which admit an event horizon, we find solutions to vacuum Einstein's equations in arbitrary dimension which have a globally null Killing field and rotational symmetry. We show that while such solutions can be deformed to include ones which are asymptotically plane wave, they do not posses a regular event horizon. If we allow for additional matter, such as in supergravity theories, we show that it is possible to have extremal solutions with globally null Killing field, a regular horizon, and which, in addition, are asymptotically plane wave. In particular, we deform the extremal M2-brane solution in 11-dimensional supergravity so that it behaves asymptotically as a 10-dimensional vacuum plane wave times a real line.

Keywords: Black Holes in String Theory, Penrose limit and pp-wave background.

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Contents

1. Introduction

Plane wave spacetimes have been on the forefront of attention over the last few months, stemming from the observation that the effective dynamics of a certain sector of the $\mathcal{N} = 4$, $d = 4$ supersymmetric Yang-Mills theory with large R-charge, is encoded in the dynamics of strings propagating in the maximally supersymmetric plane wave background of Type IIB supergravity [1]. It is even more fascinating that the world-sheet sigma model is a solvable conformal field theory, despite there being non-trivial Ramond-Ramond fluxes in the background [1, 2].

An interesting variation of the BMN correspondence is to consider a thermal version of the same (cf. [3]–[5], for computations of the partition function). Naively, one would imagine that the aforementioned gauge theory system in a thermal ensemble ought to have its dynamics encoded in the propagation of strings in an asymptotically plane wave black hole background. This would be in direct analogy with the standard story in the AdS/CFT correspondence [6], wherein thermalizing the gauge theory corresponds to looking at black holes in the dual supergravity background. From this standpoint it would be very interesting to examine whether there can be black holes which are asymptotically plane wave.

Although plane waves have been known as solutions to Einstein's equations for many decades now, very little is known about these spacetimes. While a few interesting facets of information, such as the fact that the Penrose limit [7] of any spacetime is a plane wave, and some global properties such as these spacetimes not being globally hyperbolic [8], have been known for a while, many questions remain unexplored. In particular, it is only recently that the causal structures of some plane wave spacetimes were constructed [9]–[11].

In a previous paper [12], we had asked whether spacetimes which are of the pp-wave form can admit event horizons. We gave evidence that this is not possible, by demonstrating the possibility of causal communication from any point in the spacetime manifold out to "infinity". We have subsequently re-confirmed the absence of horizons [11] by considering the full causal structure of pp-waves. This basically implies that spacetimes admitting a covariantly constant null Killing field do not admit event horizons.

A natural question which then presents itself is whether one can find black hole solutions by relaxing some part of the requirement of a covariantly constant null Killing field. As a second step, we demonstrated in [12] that there exist spacetimes which admit a globally null, Killing, but not covariantly constant, vector field, which have a regular horizon, and in addition are asymptotically plane wave. The strategy was to start with an asymptotically flat spacetime with a globally null, hypersurface-orthogonal Killing field which admits a regular horizon, and deform it to be asymptotically plane wave by using the Garfinkle-Vachaspati method [13, 14]. In particular, we exhibited a five-dimensional charged black string solution [15] which had the requisite properties, and deformed it to be asymptotically plane wave.

We would like to enquire whether there are analogous neutral black string solutions which may be asymptotically plane wave. We saw in [12] that one cannot simply "uncharge" the solution by modifying the parameters, because then the horizon shrinks to zero size and becomes singular. So, instead, we explore whether we can actually find solutions to vacuum Einstein's equations by directly solving them for a general metric ansatz with the appropriate symmetries.

The general question we pose is: *Do there exist neutral black string solutions in vacuum gravity which admit a globally null Killing field and are asymptotically plane wave?* Making the additional requirement that the solution be rotationally symmetric, we find that such solutions do *not* exist. In particular, we find all solutions to vacuum Einstein's equations, with a globally defined null Killing field and rotational symmetry in the transverse plane. Although we can subsequently deform these to break the rotational symmetry and induce the correct asymptotics, none of such solutions can correspond to a black string, i.e. , they don't admit horizons.

We find that there are two classes of solutions: one, which we shall discuss below in some detail, which is asymptotically flat; and the other, described by pure pp-waves. The latter, as we have demonstrated [12], does not admit horizons, i.e., it cannot represent the black string, albeit having the correct asymptopia in case of plane waves. On the other hand, the former is asymptotically flat; but it can be deformed to be asymptotically plane wave using the Garfinkle-Vachaspati method [13, 14]. However, these solutions have naked singularities, so unless we can resolve the singularity and cloak it with an appropriate event horizon, they would not correspond to the solutions we are looking for.

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This is a quite a remarkable result. Restricting ourselves to vacuum Einstein's equations severely restricts the nature of the possible solutions, and in particular eliminates the interesting black string ones. Thus, to obtain a neutral black object in asymptotically plane wave spacetime, we might be forced to forgo some of the symmetries we imposed above. In particular, if there is no globally null Killing field, one can imagine patching the ordinary vacuum black hole/string into the plane wave. We leave this for future consideration.

Given the absence of solutions to vacuum Einstein's equations with a globally null Killing field and a regular horizon, we might ask about the corresponding status in supergravity. There the story is much richer and more promising. An interesting class of solutions corresponds to the extremal black brane solutions in Type II supergravity in ten dimensions or in eleven dimensional supergravity. As we will explicitly demonstrate below, all the Dp branes with $p \geq 1$ and the M-brane solutions can be easily deformed to be asymptotically plane wave. The most interesting case is that of the M2-brane, since it is the only solution of the above class which admits a regular horizon with a singularity behind. This solution may be trivially deformed to one which is asymptotically a ten dimensional vacuum plane wave[1] times a real line i.e., $\mathcal{V}_{10} \times \mathbb{R}$.

The organization of the paper is as follows. We begin in section 2 with a discussion of the allowed solutions to vacuum Einstein's equations with the requisite symmetries. These will be the new solutions which we present in this work. For clarity of presentation, we will first discuss solutions which have an additional isometry and then show how one might consider more general solutions. After a brief discussion of their properties, we turn to a brief review of the Garfinkle-Vachaspati technique in section 3, and demonstrate how it may be used to convert our vacuum solutions to have plane wave asymptotics. In section 4, we turn to supersymmetric solutions and show how to construct interesting spacetimes which have an asymptotic plane wave structure. We end with a discussion in section 5.

2. Vacuum solutions

In the present section, we will write down all solutions of vacuum Einstein's equations with a globally null Killing field and rotational symmetry. The solutions we find fall into two classes:

a. pp-waves, which have a covariantly constant null Killing field.

b. Solutions with a null Killing field which is not covariantly constant. These solutions are singular and asymptotically flat.

In order to simplify the discussion, we begin by demanding an additional isometry. As we shall show later, it is possible to write down more general solutions by relaxing this requirement.

2.1 Ansatz

We first write down the metric ansatz for the spacetime admitting all the required symmetries. A naive guess would be to write down an ansatz for the metric motivated by the

[1]In what follows, we will denote the vacuum plane wave in d-dimensions as $\mathcal{V}_d$

form of the extremal black string solution presented in [12] and [15]–[17]. The solution was given as

$$ds^2_{\rm str} = \frac{2}{h(r)}\, du\, dv + \frac{f(r)}{h(r)}\, du^2 + k(r)\, l(r) \left(dr^2 + r^2 d\Omega_2^2\right), \tag{2.1}$$

which describes the metric in the string frame. However, as the dilaton depends only on the radial coordinate, the form of the solution remains the same in the Einstein frame. So we can write our metric ansatz as[2]

$$ds^2 = \frac{1}{H(r)} \left(-2\, du\, dv + F(r)\, du^2\right) + G(r)\, (dr^2 + r^2\, d\Omega_{d-3}^2). \tag{2.2}$$

In this section we will concern ourselves with purely the Einstein-Hilbert action with no matter. Given the absence of massless scalar fields, we have no ambiguity in the choice of frame, i.e., the above metric is the Einstein frame metric.

This is in fact, up to diffeomorphisms, the most general form of a metric satisfying the following properties: it has a globally null Killing field, $(\partial/\partial v)^a$, apart from possessing the additional Killing field $(\partial/\partial u)^a$, and being spherically symmetric in the plane transverse to these two directions. The metric on these spheres given by $d\Omega_{d-3}^2$, and the remaining radial coordinate is denoted by r.

In order to see that this is indeed the minimal ansatz with the requisite symmetry, realize that we have effectively a four-dimensional metric in the coordinates (u, v, r, Ω), having fixed the metric on the spheres. This implies ten components of the metric. However, using the fact that the metric has a null symmetry and a rotational invariance, we may set g_{vv} and $g_{i\Omega}$ to zero for $i = (u, v, r)$. We might also fix the radial coordinate, so that the spheres of radius r have the appropriate area. This implies five independent functions and the metric takes the form

$$ds^2 = -2\, A(r)\, du\, dv + B(r)\, du^2 + 2\, C(r)\, dv\, dr + 2\, D(r)\, du\, dr + E(r)\, dr^2 + r^2\, d\Omega_{d-3}^2\,. \tag{2.3}$$

One can now define new coordinates by first choosing $d\tilde{u} = du - (C(r)/A(r))\, dr$ and then $\tilde{v}$, satisfying $d\tilde{v} = dv - (1/A(r))\, (D(r) + B(r)\, C(r)/A(r))\, dr$. Then the metric reduces to the canonical form given in (2.2), after some trivial redefinition of coordinates.

2.2 General solution

Solving the vacuum Einstein's equations in d dimensions with the ansatz (2.2) yields 3 classes of solutions. First, we have the solution

$$G(r) = G_0 = \text{const.}\,, \qquad H(r) = H_0 = \text{const.}\,, \tag{2.4}$$

and $F(r)$ is harmonic in the transverse coordinate, namely $F(r) = F_0 + F_1/r^{d-4}$ where F_0 and F_1 are constants. But these are just the usual vacuum pp-waves. We can see this explicitly by fixing $H_0 \equiv 1$ by appropriate constant rescaling of u and v, and similarly $G_0 \equiv 1$ by rescaling r. The metric (2.2) then becomes

$$ds^2 = -2\, du\, dv + F(r)\, du^2 + dr^2 + r^2\, d\Omega^2 \tag{2.5}$$

which is just the pp-wave metric.

[2] In the following, we redefine $v \to -v$, to keep the notation consistent with [11] and the bulk of [12].

The second class looks slightly less trivial,

$$G(r) = \frac{\alpha}{r^4}\,, \qquad H(r) = H_0 = \text{const.}\,, \tag{2.6}$$

and again $F(r)$ is harmonic. But these are in fact identical to the previous solutions (2.4). To see that, consider the coordinate transformation $r \to 1/r$. Fixing the constants appropriately, we obtain (2.5).

Finally, the third and most interesting class of solutions in $d > 4$ dimensions is given by

$$\begin{aligned} ds^2 &= \frac{1}{H(r)}\left(-2\,du\,dv + F(r)\,du^2\right) + G(r)\left(dr^2 + r^2\,d\Omega_{d-3}^2\right) && (2.7)\\ H(r) &= c_1\left(\frac{r^{d-4} - a}{r^{d-4} + a}\right)^{\sqrt{2(d-3)/(d-2)}} && (2.8)\\ G(r) &= c_2\left(\frac{H(r)^2}{H'(r)\,r^{d-3}}\right)^{2/(d-4)} && (2.9)\\ F(r) &= c_3 + c_4 \ln H(r)\,, && (2.10)\end{aligned}$$

where a, c_1, c_2, c_3, and c_4 are arbitrary constants. For the metric to have the correct signature, we require that $c_1 > 0$. In fact, we can fix many of the constants to be $\equiv \pm 1$ by appropriate diffeomorphisms. For example, we can fix $c_3 \equiv 1$ by rescaling u and absorbing c_3 in c_4 and c_1, then $c_1 \equiv 1$ by rescaling u and v, and $c_2 \equiv 1$ by rescaling r and a. Whether $(\partial/\partial u)^a$ is a timelike or spacelike (or null) Killing field depends on the sign on $F(r)/H(r)$, in particular, on the constants involved. For $c_1 > 0$, if both c_4 and c_3 in (2.7) have the same sign, then $(\partial/\partial u)^a$ is null at some r, whereas if $c_3 < 0 < c_4$, $(\partial/\partial u)^a$ is globally timelike and if $c_4 < 0 < c_3$, it is globally spacelike. (If $c_4 = 0$, we in fact have two globally null Killing fields.) We can rewrite G a bit more explicitly (with an appropriate choice of c_2 to absorb the d-dependent coefficient) as

$$G(r) = \left(H(r)\,\frac{(r^{d-4} - a)(r^{d-4} + a)}{r^{2d-8}}\right)^{2/(d-4)}\,. \tag{2.11}$$

The solution in the special case of four dimensions is given as

$$ds_4^2 = (1 + \ln r)\,\left[-2\,du\,dv + (1 + \ln(1 + \ln r))\,du^2\right] + \frac{1}{r^2\sqrt{1 + \ln r}}\left(dr^2 + r^2 d\theta^2\right). \tag{2.12}$$

In writing the above we have made a particular choice of the constants of integration so as to present the solution in the simplest form.

2.3 Properties of the solution

Let us now focus on the last class of solutions, given by (2.7), with (2.8), (2.9), and (2.10). We see that these are asymptotically flat spacetimes, since we can fix $c_1 = c_2 = c_3 \equiv 1$, and then $H(r) \to 1$ as $r \to \infty$, so that from (2.11), $G(r) \to 1$ as $r \to \infty$, and clearly $F(r) \to 1$ as $r \to \infty$.

Also, these are explicitly distinct from pp-waves, since not all the curvature invariants vanish. In fact, the curvature invariant composed from the Weyl tensor blows up as $r^{d-4} \to$

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$\pm a$ (r is positive but a can have either sign). This is suggestive of a curvature singularity; one can in fact easily verify that these spacetimes are singular by noting that radially in-going geodesics end there at finite affine parameter. Thus, we restrict our spacetime to $r > |a|^{1/(d-4)}$.

Furthermore, from the form of the metric, in particular from (2.8) and (2.9), it seems that there are no event horizons. A quick way to verify the absence of horizons is to show that there are causal curves from any point of the spacetime which can communicate 'out to infinity'. As we noted in [12], for any solution with a null Killing field $(\partial/\partial v)^a$, the orbits of the Killing field actually describe null geodesics; so we can always reach $v \to \infty$ by a null geodesic. However, this does not suffice, since we also want to show causal communication with large transverse directions. To this end, we will present a causal (in fact a null) curve which reaches $r \to \infty$. Consider in the metric (2.7) a null curve $\mathcal{C}$ parameterized by λ, with the coordinates along $\mathcal{C}$ satisfying

$$\begin{aligned} u(\lambda) &= \lambda \\ v(\lambda) &= \alpha\,\lambda \\ \frac{d\Omega}{d\lambda} &= 0 \\ \left(\frac{dr}{d\lambda}\right)^2 &= \frac{2\alpha - F(r)}{G(r)\,H(r)}\,. \end{aligned} \tag{2.13}$$

From the explicit functional behaviour of $F(r)$, $G(r)$, and $H(r)$, it is clear that $\mathcal{C}$ exists from any point with $r > a^{1/(d-4)}$ as long as we make a judicious choice of $\alpha > 0$. In particular, since one can canonically choose $F(r) = 1 + c_4 \ln H(r)$, by choice of $2\alpha \geq 1 + c_4 \ln(H(r = a^{1/(d-4)} + \varepsilon))$, for arbitrarily small ε, we can achieve our desired objective of communicating to the asymptotic regions. For $c_4 > 0$, it suffices to let $\alpha > 1/2$. Note that we can also use the same strategy as in [12] to "bound" the spacetime (2.7) by a flat space metric, and use causality arguments to show that there are no horizons.

One can write the metric (2.7) in a more suggestive form, by a change of coordinates. Let us denote $\alpha = \sqrt{2(d-3)/(d-2)}$ and $\beta = 2/(d-4)$. Then in dimensions $d \geq 5$ choosing $r^{d-4} = a\,e^{2x}$ we find that we can cast the metric in the form

$$ds^2 = \frac{1}{(\tanh x)^\alpha}\Big(- 2\,du\,dv + (1 + \ln\tanh x)\;du^2\Big) + (\tanh x)^{\alpha\beta}\,(\sinh 2x)^\beta\,\Big(\beta^2\,dx^2 + d\Omega^2\Big)\,. \tag{2.14}$$

Here the coordinate $x \in (0, \infty)$ and the singularity is at $x = 0$. For all values of $x > 0$, the metric is everywhere smooth, and one doesn't expect there to be any horizons.

In the four dimensional case writing, $\rho^2 = 1 + \ln r$, we obtain

$$ds^2 = \frac{1}{\rho^4}\left(-2\,du\,dv - \ln\rho^4\,du^2\right) + 16\,\frac{1}{\rho^8}\,d\rho^2 + \rho^2\,d\theta^2\,. \tag{2.15}$$

Clearly, this is not asymptotically flat. In fact, so much may be inferred from the original metric in (2.12). This had to be the case, for in four dimensional spacetime, the transverse space being two-dimensional, causes the asymptotics to change quite drastically, leading to logarithmic deviations from flat space. We will not have much to say about this solution in what follows.

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2.4 Abandoning the $(\partial/\partial u)^a$ isometry

We now come to the interesting case, when $(\partial/\partial u)^a$ is no longer an isometry. In this case, one might consider the situation wherein we allow all the functions appearing in (2.2) to have dependence on the u coordinate as well. So we look for spacetimes with a metric ansatz

$$ds^2 = \frac{1}{H(u,r)} \left(-2\, du\, dv + F(u,r)\, du^2\right) + G(u,r)\, (dr^2 + r^2\, d\Omega_{d-3}^2) + 2A(u,r)\, du\, dr\,. \quad (2.16)$$

In fact, we can show that the last term, $2A(u,r)\, du\, dr$, can be set to zero by appropriate coordinate transformation on v. The simplest way to break the $(\partial/\partial u)^a$ isometry is to just have the constants of integration in function $F(u,r)$ appearing in (2.10) to be functions of u, i.e., choose $F(u,r) = f_1(u) + f_2(u) \ln H(r)$. The functions $H(r)$ and $G(r)$ are given as before in (2.8), (2.9). This is very similar to the fashion in which one breaks the analogous isometry in pp-waves.

The most general solutions to the metric ansatz (2.16) can also be found, and the functions appearing in the ansatz are given as:

$$\begin{aligned} H(u,r) &= h_1(u) \left(\frac{r^{d-4} - h_2(u)}{r^{d-4} + h_2(u)}\right)^{\sqrt{2(d-3)/(d-2)}} \\ G(u,r) &= g(u) \left(\frac{H(u,r)^2}{\partial_r H(u,r)\, r^{d-3}}\right)^{2/(d-4)} \\ F(u,r) &= f_1(u) + f_2(u)\, \ln H(u,r)\,. \end{aligned} \quad (2.17)$$

The solution in addition is characterized by four arbitrary functions $f_1(u)$, $f_2(u)$, $h_1(u)$ and $h_2(u)$ which are unconstrained. The function $g(u)$ is given in terms of $h_1(u)$ as $g(u) = h_1(u)^{-2/(d-4)}$.

Before ending this section, let us make a brief comment on the rotational symmetry. All the solutions presented above were by construction rotationally symmetric in the transverse directions. Since no nontrivial vacuum plane wave can preserve this symmetry, we could not have generated vacuum solutions which would be asymptotically plane wave. (Note that this is no longer true if we relax the vacuum requirement.) Indeed, as discovered above, all our solutions were asymptotically flat, which is consistent, since flat spacetime is the only rotationally symmetric vacuum plane wave. However, as we will demonstrate explicitly in the next section, we can deform these solutions in such a way as to break the rotational symmetry and at the same time lift the asymptotic behavior to be that of a vacuum plane wave.

While this method generates the requisite asymptopia, it does not modify the structure near the singularity sufficiently to generate a horizon. We will demonstrate this feature for an explicit example. However, despite having broken the rotational symmetry, the metric thus generated is still not the most general non-rotationally symmetric spacetime. In particular, one may ask whether one could not first break the rotational symmetry so as to obtain an asymptotically flat solution with horizons, and then use the solution-generating technique discussed below to make the solution have plane wave asymptopia.

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Whereas this is difficult to check analytically, we believe that such a scenario is unlikely, because of general properties of black holes. Specifically, one would expect that if a non-spherical solution exists, we could obtain a spherically symmetric one by "smearing" or "superposing" these solutions over all directions appropriately. Intuitively, one might think of colliding the nonsymmetric black holes so as to effectively restore the symmetry; and by the area theorem, the final configuration should still possess a horizon. Hence, the lack of such a symmetric black hole would naively suggest the nonexistence of any non-spherically symmetric one.

3. Deforming the solutions

The vacuum solutions we wrote down in section 2.2 are asymptotically flat. We are really interested in solutions which are asymptotically plane wave. Given an asymptotically flat solution we can use the Garfinkle-Vachaspati construction to deform it to be asymptotically plane wave. We begin by reviewing the construction in a more general (non-vacuum) setup, and then proceed to apply the same to the vacuum solutions discussed in section 2.

3.1 Review of the Garfinkle-Vachaspati construction

The Garfinkle-Vachaspati (GV) construction [13, 14], is essentially a solution generating technique. Given a solution to Einstein's equations (in general, with some appropriate matter content), with certain specific properties, one can deform the solution to a new one with the same matter fields. In particular, the scalar curvature invariants of the deformed solution are identical to the parent solution. The idea is that given a solution with an appropriate set of symmetries, one can essentially "linearize" Einstein's equations, which will then allow one to superpose solutions. This technique was developed in an attempt to add wavy hair to black holes/strings. We will in the following briefly review the construction as presented in [15].

We assume that we are working with the Einstein-Hilbert action with some matter fields. To be specific, we assume that the matter content is the conventional matter appropriate for supergravity theories. In particular, we will have scalar fields ϕ_i, and a bunch of $p-$form fields A_p with their associated field strengths $F_{(p+1)}$. We will work with the action,

$$S = \int d^d x \sqrt{-g} \left(R - \frac{1}{2} \sum_i \alpha_i(\phi)\ (\nabla \phi_i)^2 - \frac{1}{2} \sum_p \beta_p(\phi)\, F^2_{(p+1)} \right) . \qquad (3.1)$$

Thus, the metric we work with will always be the Einstein frame metric, although one might generalize the discussion to work with the string frame metric. The couplings $\alpha_i(\phi)$ and $\beta_p(\phi)$ are assumed to be non-derivative couplings for convenience.

The primary requirement for a solution, $\mathcal{M}(g_{\mu\nu}, \phi_i, F_{(p+1)})$, to be amenable to the GV construction is that it possess a null, hypersurface-orthogonal Killing field, i.e., the solution

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admits a vector field k^μ which satisfies

$$\begin{aligned} k_\mu\, k^\mu &= 0 \\ \nabla_{(\mu} k_{\nu)} &= 0 \\ \nabla_{[\mu} k_{\nu]} &= k_{[\mu} \nabla_{\nu]} S\,. \end{aligned} \tag{3.2}$$

The first two conditions demand that the vector be null and Killing, while the last condition is that of hypersurface-orthogonality. S is a scalar function on the background geometry. Note that if S is a constant, then the vector k^μ must be covariantly constant. This is the case for pp-waves.

With respect to the matter fields supporting the metric in $\mathcal{M}(g_{\mu\nu},\, \phi_i,\, F_{(p+1)})$, we require that the scalar fields have a vanishing Lie derivative with respect to the vector k^μ i.e., $\mathcal{L}_k\, \phi_i = 0$. We impose the same constraint on the $(p+1)-$form field strengths; $\mathcal{L}_k\, F_{(p+1)} = 0$. However, it will turn out that in order to maintain the form of the field strength in the deformed solution, we will require further that they satisfy an additional transversality condition $i_k\, F_{(p+1)} = \omega_k \wedge \theta_{(p-1)}$. Here, i_k denotes the interior product, while ω_k is the one-form associated with the vector k^μ, and $\theta_{(p-1)}$ is some $(p-1)-$form. These requirements are necessitated to ensure that upon deforming the metric, the change in stress tensor, given the functional form of the matter fields in $\mathcal{M}(g_{\mu\nu},\, \phi_i,\, F_{(p+1)})$, is such that one is left with a linear equation for the deformation parameter.

Given $\mathcal{M}(g_{\mu\nu},\, \phi_i,\, F_{(p+1)})$ as a solution to (3.1) with the above properties, we can find a new solution $\hat{\mathcal{M}}(G_{\mu\nu},\, \phi_i,\, F_{(p+1)})$, where the new metric $G_{\mu\nu}$ is defined in terms of a deformation Ψ as follows:

$$G_{\mu\nu} = g_{\mu\nu} + e^S\, \Psi\, k_\mu\, k_\nu\,. \tag{3.3}$$

As the notation suggests, the form of the matter fields is left unchanged. The function Ψ satisfies the following constraints

$$k^\mu\, \nabla_\mu\, \Psi = 0\,, \qquad \text{and} \qquad \nabla^2\, \Psi = 0\,. \tag{3.4}$$

The first of these is equivalent to demanding that Ψ has vanishing Lie derivative along k^μ, and the second just demands that Ψ solve the covariant Laplace equation in the original background, i.e., the laplacian with respect to the metric $g_{\mu\nu}$. It may be verified that the deformed solution retains the scalar curvature invariants of the parent solution. For further details of the construction and proof of the equivalence of the scalar curvature invariants the reader is referred to e.g. [15].

As a simple example of the GV construction, consider a plane wave spacetime, with the metric

$$ds^2 = -2\, du\, dv - f_{ij}(u)\, x^i\, x^j\, du^2 + (dx^i)^2\,. \tag{3.5}$$

We assume that the metric is supported by an appropriate stress tensor, which goes into the determination of the precise form of the functions $f_{ij}(u)$. Now, the Einstein's equations are linear because of the fact that the background admits a covariantly constant null Killing field $(\partial/\partial v)^a$. Consider now the metric

$$ds^2 = -2\, du\, dv - \big(f_{ij}(u)\, x^i\, x^j - F(u, x^i)\big)\; du^2 + (dx^i)^2\,. \tag{3.6}$$

This metric will satisfy the equations of motion with the same matter content, so long as $F(u, x^i)$ is harmonic in the transverse dimensions x^i. (In terms of (3.3), $S \equiv 0$ and $\Psi \equiv F$.) But (3.6) has the general form of a pp-wave spacetime. Thus, one trivial application of the GV construction is to deform any plane wave spacetime into a pp-wave spacetime.

3.2 Vacuum solutions and Garfinkle-Vachaspati construction

In the present subsection we will apply the GV construction to the class of solutions presented in section 2. In this case, since we have no matter fields present, the discussion is much simplified. All we need to do is to ensure that the metric satisfies the appropriate constraints as in (3.2) and to find suitable deformations of the same.

Let us start with the form of the metric as in (2.2), wherein for simplicity of discussion we will revert back the case with the extra isometry. As mentioned earlier, it is clear that $(\partial/\partial v)^a$ is a null Killing field. To check that it in addition is hypersurface-orthogonal is trivial and one finds that $S = \ln H(r)$. So the new metric according to (3.3) will in general be of the form

$$ds^2 = \frac{1}{H(r)} \left(-2\, du\, dv + F(r)\, du^2\right) + G(r)\, (dr^2 + r^2\, d\Omega_{d-3}^2) + \frac{1}{H(r)}\, \Psi(u, v, r, \Omega_{d-3})\, du^2\,. \tag{3.7}$$

The first of the conditions in (3.4) implies that $\partial_v\, \Psi(u, v, r, \Omega_{d-3}) = 0$ and therefore the second implies that $\Psi(u, r, \Omega_{d-3})$ satisfy:

$$\nabla^2\, \Psi = \frac{H(r)}{r^{d-3}\, G(r)^{(d-2)/2}}\, \frac{d}{dr} \left(\frac{G(r)^{(d-4)/2}\, r^{d-3}}{H(r)}\, \frac{d\Psi}{dr} \right) + \frac{1}{r^2 G(r)}\, \nabla^2_{\Omega_{d-3}}\, \Psi = 0\,. \tag{3.8}$$

Decomposing $\Psi(u, r, \Omega_{d-3})$ into spherical harmonics on the $\mathbf{S}^{d-3}$, labeled by $L = (\ell, m_1, \ldots)$, with principal angular momentum ℓ, the general solution takes the form:

$$\Psi(u, r, \Omega_{d-3}) = \sum_L\, \xi_L(u)\, \psi_\ell(r)\, Y_L(\Omega_{d-3})\,. \tag{3.9}$$

The function $\psi_\ell(r)$ satisfies a one-dimensional radial wave equation

$$\frac{H(r)}{r^{d-3}\, G(r)^{(d-4)/2}}\, \frac{d}{dr} \left(\frac{G(r)^{(d-4)/2}\, r^{d-3}}{H(r)}\, \frac{d\psi_\ell(r)}{dr} \right) - \frac{\ell(\ell + d - 4)}{r^2}\, \psi_\ell(r) = 0\,. \tag{3.10}$$

Solutions of this equation will determine the generically allowed deformations of our vacuum solutions. Using the relation between the functions $G(r)$ and $H(r)$ as in (2.9), this can be simplified to read

$$\frac{H'(r)}{H(r)}\, \frac{d}{dr} \left(\frac{H(r)}{H'(r)}\, \frac{d\psi_\ell(r)}{dr} \right) - \frac{\ell(\ell + d - 4)}{r^2}\, \psi_\ell(r) = 0\,. \tag{3.11}$$

However, we are interested in the particular case when asymptotically the solution is of the plane wave form. To determine if this is possible, all we need is to examine the asymptotics of (3.11) and demand that the solution behave like r^2. In general, given that the functions $H(r)$ and $G(r)$ tend to one as $r \to \infty$, in dimensions $d \geq 5$, we see that the asymptotics of the solution for Ψ is the same as a harmonic function in $(d-2)$ dimensions. So we are guaranteed to have a solution wherein $\Psi(u, r \to \infty, \Omega_{d-3}) \to r^2\, Y_2(\Omega_{d-3})$. We therefore need to solve the radial equation (3.11) for the $\ell = 2$ mode in order to find solutions that are asymptotically plane wave.

As an example let us consider the vacuum solution (2.7) in five dimensions. The resulting metric is explicitly given as

$$\begin{aligned} ds^2 &= \frac{1}{H(r)}\left(-2\, du\, dv + \left[1 + \ln H(r) + \xi_2(u)\, \psi_2(r)\, \left(3\cos^2\theta - 1\right)\right]\, du^2 \right) + \\ &\quad + \frac{H(r)^4}{r^4\, H'(r)^2}\left(dr^2 + r^2\, d\theta^2 + r^2\, \sin^2\theta\, d\phi^2 \right) \\ H(r) &= \left(\frac{r-1}{r+1}\right)^{2/\sqrt{3}} \\ \psi_2(r) &= \left(3r^2 + 2 + \frac{3}{r^2}\right)\, \left(\alpha_1 + \alpha_2\, \ln\left(\frac{r-1}{r+1}\right)\right) + 6\alpha_2\, \left(r + \frac{1}{r}\right) \end{aligned} \tag{3.12}$$

with arbitrary integration constants α_1 and α_2 and as before $\xi_2(u)$ is an arbitrary function of u. The asymptotic behaviour of the solution is clearly that of a vacuum plane wave in five dimensions, $\mathcal{V}_5$.

4. Supersymmetric solutions

In section 2, we saw that vacuum solutions to Einstein's equations with a globally null Killing field and transverse rotational invariance, are uninteresting from the point of view of being black string solutions with a regular horizon. Nonetheless, these can be deformed to be asymptotically plane wave, notwithstanding the fact that we are still eluded from our goal of finding asymptotically plane wave black strings. One might be tempted to claim that the requirement of a global null symmetry is too restrictive to allow for black holes/strings. However, the fallacy of this logic is all too apparent when one considers solutions to Einstein's equations with appropriate matter content. In particular, the solution presented in [12] did possess a globally null Killing field and was a solution to the low energy effective action of the five dimensional heterotic string.

In supergravity theories we can easily find many examples of solutions which admit a globally null Killing field and in addition have regular horizons. To illustrate this point, it is worthwhile to consider the most famous supersymmetric solutions, the extremal black branes in ten (IIA/IIB) or eleven dimensional supergravity. In eleven dimensional supergravity we have the extremal branes, and in IIA (IIB) supergravity we have the even (odd) Dp-brane solutions, all of which are asymptotically flat. Since the extremal black brane solutions have an isometry group $SO(1,p) \times SO(9-p)$, for $p > 1$ it is clear that we have always two null isometries in the solution. Given two everywhere null Killing fields, we can deform the solution using the GV construction and make it asymptotically plane wave.

Of these solutions, the most interesting are the D3, M2 and M5 branes. The extremal D3 and the M5 brane solutions are asymptotically flat and admit a maximally extended

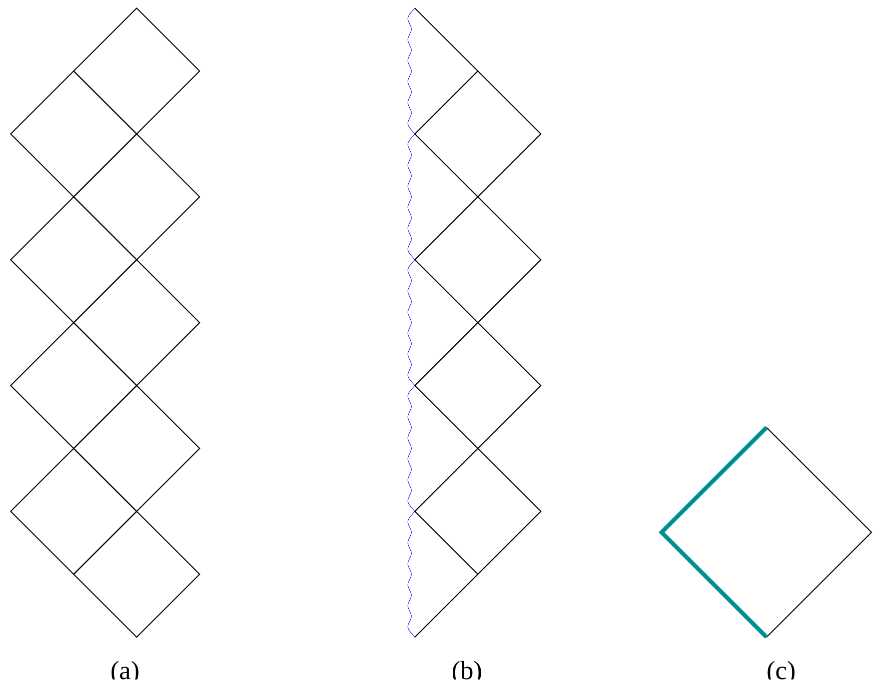

Figure 1: Penrose diagrams of the extremal black branes. (a) corresponds to the M5-brane and D3-brane, (b) corresponds to the M2-brane, and (c) denotes the other Dp-branes for $p < 6$ and $p \neq 3$.

spacetime which is everywhere smooth and has regular horizons. The near-horizon geometry is of course the well known $AdS_5 \times \mathbf{S}^5$ spacetime for D3 branes and $AdS_7 \times \mathbf{S}^4$ for the M5 brane. In the case of the M2-brane, the horizon cloaks a timelike singularity, and the causal structure of the maximally-extended spacetime looks like that of the extremal Reissner-Nordström black hole in four dimensions. The near horizon geometry is of course $AdS_4 \times \mathbf{S}^7$. The remaining D$p$-brane solutions for $p \leq 6$ have the singularity coincident with the horizon and in this sense do not posses a regular horizon. The Penrose diagrams for the various brane solutions are given in figure 1.

Thus, while all the D-branes are in principle candidates for deformation into an asymptotically plane wave geometry using the GV construction, only the M2-brane solution is capable of producing a spacetime which has in addition a regular event horizon cloaking a singularity. While the GV construction does not guarantee that the causal structure of the original and the deformed spacetimes will remain the same, the curvature invariants of the two metrics are identical. Therefore, for general Dp-brane solutions, the deformation by the GV construction to make the spacetime asymptotically plane wave will not alleviate the singular property of the horizon. For the case of the non-dilatonic branes with a regular event horizon, we will see that the term added to implement the GV construction will be vanishingly small in the vicinity of the horizon, thereby preventing the horizon from being destabilized in the resulting spacetime (for an explicit check, see the analysis in [15] for the 5-dimensional black hole case). So in what follows, we shall dwell mostly on the case of the M2-brane and only briefly mention the other cases.

4.1 Deforming the M2-brane

In the following we show that it is possible to deform the M2-brane solution in eleven dimensional supergravity so that the asymptotic behaviour is $\mathcal{V}_{10} \times \mathbb{R}$, whilst retaining the nature of the near-horizon geometry and the singularity intact. For a discussion of supergravity solutions of Dp-branes in plane wave backgrounds, which give rise to similar metrics see [18].

The M2-brane solution is given as

$$\begin{aligned} ds^2 &= H_2(r)^{-2/3} \left(-dt^2 + dx_1^2 + dx_2^2\right) + H_2(r)^{1/3} \left(dr^2 + r^2\, d\Omega_7^2\right) \\ F_4 &= \left(\frac{dH_2(r)^{-1}}{dr}\right) dt \wedge dx_1 \wedge dx_2 \wedge dr\,, \end{aligned} \tag{4.1}$$

where $H_2(r) = 1 + Q^6/r^6$ is a harmonic function in the transverse eight-dimensional space. The horizon is at $r = 0$ in these coordinates. To see the location of the singularity, it is best to define a new coordinate $\zeta = r^2$. The singularity is located at the zero of $H(\zeta)$, i.e., $\zeta = -Q^2$.

We can define two null directions u, v by linear combinations of t and x_1 and write the solution as

$$\begin{aligned} ds^2 &= H_2(r)^{-2/3} \left(-2\, du\, dv + dx_2^2\right) + H_2(r)^{1/3} \left(dr^2 + r^2\, d\Omega_7^2\right) \\ F_4 &= \left(\frac{dH_2(r)^{-1}}{dr}\right) du \wedge dv \wedge dx_2 \wedge dr\,. \end{aligned} \tag{4.2}$$

Now, both $(\partial/\partial v)^a$ and $(\partial/\partial u)^a$ are null Killing vectors and we can also verify that they are hypersurface-orthogonal. Moreover, the field strength satisfies the requisite transversality condition. In short, we are allowed to deform the solution using the GV construction by choosing one of the two null Killing vectors. To wit, we can write the deformed metric as

$$ds^2 = H_2(r)^{-2/3} \left(-2\, du\, dv + dx_2^2 - \Psi(u, x_2, r, \Omega_7)\, du^2\right) + H_2(r)^{1/3} \left(dr^2 + r^2\, d\Omega_7^2\right). \tag{4.3}$$

We have already taken into account the first of the constraints on Ψ in (3.4), by having $\partial_v \Psi = 0$. The second constraint reduces to

$$H_2^{1/3}(r)\, \nabla^2\, \Psi = H_2(r) \frac{d^2\Psi}{dx_2^2} + \frac{d^2\Psi}{dr^2} + \frac{7}{r}\frac{d\Psi}{dr} + \frac{1}{r^2} \nabla^2_{\Omega_7}\, \Psi = 0\,. \tag{4.4}$$

Writing $\Psi(u, x_2, r, \Omega_7) = \xi_{kL}(u)\, e^{ikx_2}\, \psi_{k\ell}(r)\, Y_L(\Omega_7)$ with arbitrary functions $\xi_{kL}(u)$, and $Y_L(\Omega_7)$ denoting the spherical harmonics with L being a label for the set of angular momenta on the seven sphere with principal angular momentum ℓ, we find the radial equation

$$\frac{d^2\psi_{k\ell}(r)}{dr^2} + \frac{7}{r}\frac{d\psi_{k\ell}(r)}{dr} - \left(\frac{\ell(\ell+6)}{r^2} + k^2\, H_2(r)\right) \psi_{k\ell}(r) = 0\,. \tag{4.5}$$

For the case of $k = 0$, i.e., requiring $(\partial/\partial x_2)^a$ to be a Killing vector in the deformed geometry, we are reduced to solving the Laplace equation in eight dimensional flat space. So we can pick the $\ell = 2$ mode on the seven sphere to obtain a solution which is asymptotically plane wave.

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Let us parametrize the seven sphere by the coordinates such that θ corresponds to the azimuthal angle, and choose $\Psi(u, x_2, r, \Omega_7) = r^2\,(8\cos^2\theta - 1)$. Then we claim that

$$
\begin{aligned}
ds^2 &= H_2(r)^{-2/3}\,\left[-2\,du\,dv + dx_2^2 - r^2\left(8\,\cos^2\theta - 1\right)\,du^2\right] + \\
&\quad + H_2(r)^{1/3}\,\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\Omega_6^2\right)\right] \\
F_4 &= \left(\frac{dH_2(r)^{-1}}{dr}\right)\,du\wedge dv\wedge dx_2\wedge dr\,,
\end{aligned}
\tag{4.6}
$$

is a solution to eleven dimensional supergravity, with the same curvature invariants as the M2-brane solution. In particular, the solution still has a regular horizon at $r = 0$ (one can also check the regularity of the horizon by looking at the near-horizon geometry). This is to be contrasted with added pp-wave like terms $\Psi(u, r) \sim \frac{1}{r}$, or having plane waves along the longitudinal directions of the brane, wherein one does encounter singularities at the horizon [19] (cf. [15, 17] and [20]–[25] for discussions of similar issues in the case of black strings/branes). Its asymptotic behaviour is that of a ten dimensional vacuum plane wave times an extra real line parameterized by x_2. Since the curvature invariants in (4.6) are identical to those in the parent M2-brane solution (4.1), we see that the singularity at $\zeta = -Q^2$ remains unchanged.

In the event of $k \neq 0$ we see that the nature of the asymptotics can be changed quite dramatically. The asymptotic behaviour of the equation (4.5) can be analyzed to show that the solutions are Bessel functions which either diverge or decay faster than r^2. So by looking for solutions wherein we have some momentum along the brane world-volume directions we do not obtain a solution that looks like a plane-wave. Thus, in order to obtain a plane wave solution, our only choice is to use the solution presented in (4.6), wherein we have an additional isometry corresponding to translations along $(\partial/\partial x_2)^a$.

It is tempting to ask whether it is not possible to deform the M2-brane solution, so that the asymptotic solution is that of the maximally supersymmetric plane wave in IIB supergravity (BMN plane wave). To achieve this we want the function $\Psi(u, x_2, r, \Omega_7) = r^2$. Let us therefore consider the following metric:

$$
ds^2 = H_2(r)^{-2/3}\,\left[-2\,du\,dv + dx_2^2 - r^2\,du^2\right] + H_2(r)^{1/3}\,\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\Omega_6^2\right)\right]. \tag{4.7}
$$

Given this metric one can readily calculate the stress tensor $T_{\mu\nu}$ supporting the solution. Writing $T_{\mu\nu} = T^{(F_4)}_{\mu\nu} + \tilde{T}_{\mu\nu}$, with $T^{(F_4)}_{\mu\nu}$ being the contribution to the stress tensor from the background 4-form field strength, $F_4 = (dH_2(r)^{-1}/dr)\,du\wedge dv\wedge dx_2\wedge dr$, and $\tilde{T}_{\mu\nu}$ is the additional part of the stress tensor necessary to satisfy Einstein's equations. One finds that the only non-vanishing component of $\tilde{T}_{\mu\nu}$ is given as

$$
\tilde{T}_{uu} = \frac{8}{H_2(r)}\,. \tag{4.8}
$$

It is not possible to generate the additional piece of the stress tensor using the 4-form field strength. While one can readily write down non-vanishing components of the 4-form to generate (4.8), for instance by writing $F_{ux_2r\theta} = 4\sqrt{6}r/H_2(r)$, it is not possible to satisfy the field strength equations of motion. So we conclude that there is no consistent solution to the equations of motion of 11-dimensional supergravity with the metric given as in (4.7). One can however make (4.7) a solution if one had, in addition to the field strength, some null dust with stress tensor (4.8).

4.2 The other black branes

Let us now turn to the other extremal brane solutions. In eleven dimensional supergravity, we can consider the M5-brane solution, given by

$$ds^2 = H_5(r)^{-1/3}\,\left(-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2\right) + H_5(r)^{2/3}\,\left(dr^2 + r^2 d\Omega_4^2\right) \quad (4.9)$$

with $H_5(r) = 1 + Q_5^3/r^3$ and the field strength $F_4 \propto \omega_4$, the volume form on the transverse $\mathbf{S}^4$. Now, we can again combine t and any one of the longitudinal directions of the M5-brane, say x_1 to form two null directions, u, v. Repeating the steps as in the previous example with the M2-brane, we see that the following is a solution to eleven dimensional supergravity:

$$\begin{aligned} ds^2 \,&=\, H_5(r)^{-1/3}\,\left[-2\,du\,dv + \sum_{i=2}^{5}(dx_i)^2 - r^2\,\left(5\,\cos^2\theta - 1\right)\,du^2\right] + \\ &\quad + H_5(r)^{2/3}\,\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\Omega_3^2\right)\right]. \end{aligned} \quad (4.10)$$

The four-form field strength supporting the solution remains proportional to the volume form on the $\mathbf{S}^4$. This solution interpolates between a near-horizon geometry ($r \ll 1$) of $AdS_7 \times \mathbf{S}^4$ and a seven dimensional vacuum plane wave with four flat longitudinal directions parameterized by x_i, $i = 2, \ldots, 4$.

We can play the same game with the extremal brane solutions to ten dimensional supergravity. The extremal Dp-brane solutions, in the string frame metric, are given by

$$\begin{aligned} ds^2_{\text{str}} \,&=\, H_p(r)^{-1/2}\,\left(-dt^2 + \sum_{i=1}^{p}(dx^i)^2\right) + H_p(r)^{1/2}\,\left(dr^2 + r^2\,d\Omega_{8-p}^2\right) \\ e^{4\Phi} \,&=\, H_p(r)^{3-p} \\ F_{(p+2)} \,&=\, \left(\frac{dH_p^{-1}(r)}{dr}\right)\,dt \wedge dx^1 \wedge \cdots dx^p \wedge dr\,. \end{aligned} \quad (4.11)$$

Yet again defining u, v as linear combinations of t and x^1, we have two null directions which are conducive to applying the GV technique. Note that in addition to the form fields we also have a non-vanishing dilaton in the cases when $p \neq 3$. The dilaton, when non-vanishing, does satisfy the requirement that it have vanishing Lie derivative along the null Killing vector.

By applying the GV technique, we find new solutions to the supergravity equations of motion;

$$\begin{aligned} ds^2_{\rm str} &= H_p(r)^{-1/2}\left[-2\,du\,dv - \Psi(u,x^i,r,\Omega_{8-p})\,du^2 + \sum_{i=2}^{p}(dx^i)^2\right] + \\ &\quad + H_p(r)^{1/2}\left(dr^2 + r^2\,d\Omega^2_{8-p}\right) \\ e^{4\Phi} &= H_p(r)^{3-p} \\ F_{(p+2)} &= \left(\frac{dH_p^{-1}(r)}{dr}\right) du\wedge dv\wedge\cdots dx^p\wedge dr \end{aligned} \tag{4.12}$$

with $\Psi(u,x^i,r,\Omega_{8-p}) = \xi_{kL}(u)\,\exp(i\sum_{i=2}^{p} k_i x_i)\,\psi_{k\ell}(r)\,Y_L(\Omega_{8-p})$. Here $\xi_{kL}(u)$ are arbitrary functions of u, with $k=\{k_2,\cdots,k_p\}$, and $Y_L(\Omega_{8-p})$ are the spherical harmonics on the $\mathbf{S}^{8-p}$ labeled by $L=(\ell,m_1,\cdots)$. $\psi_{k\ell}(r)$ satisfies the radial wave equation

$$\frac{d^2\psi_{k\ell}(r)}{dr^2} + \frac{(8-p)}{r}\frac{d\psi_{k\ell}(r)}{dr} - \left(\frac{\ell(\ell+7-p)}{r^2} + \sum_{i=2}^{p}(k_i)^2\,H_p(r)\right)\psi_{k\ell}(r) = 0\,. \tag{4.13}$$

It is clear that in the case $k_i\equiv 0$, one has solutions that behave like r^2 asymptotically, for $l=2$. Thus, it is possible to deform the D$p-$brane solutions so that their asymptotic behaviour changes from $\mathbb{R}^{1,9}$ to $\mathcal{V}_{11-p}\times\mathbb{R}^{p-1}$. However, as argued earlier, D$p-$brane solutions with $p\neq 3$ do not admit a regular horizon. In the case of the D3-brane the GV construction provides an example of a solution that smoothly interpolates between a near-horizon geometry of $AdS_5\times\mathbf{S}^5$ and an asymptotic behaviour of $\mathcal{V}_8\times\mathbb{R}^2$.

5. Discussion

We have discussed solutions to vacuum Einstein's equations with a globally null Killing field and a transverse rotational symmetry. Knowledge of the explicit form of the solutions enabled us to conclude that there are no horizons in these spacetimes. While one would have thought that, in order to find vacuum asymptotically plane wave solutions, it would have been necessary to break the transverse rotational symmetry from the outset, we demonstrate that it is possible to deform the rotationally symmetric ones into the desired form.

However, the main motivation for undertaking the exercise hasn't been realized, in the sense that we haven't found a vacuum black hole solution that is asymptotically plane wave. It is clear that the requirement of a globally null Killing field is too restrictive to allow for black hole solutions in vacuum gravity. On the other hand, in theories with some matter content we are able to generate the solutions possessing regular event horizons and having the desired asymptotics. Before turning our attention to these, we note that a possible strategy at generating vacuum solutions which have horizons and the correct asymptotics is to abandon the requirement of a globally null Killing field, by considering a small g_{vv} contribution to the metric. One such possibility is to attempt to patch a small black hole solution into the vacuum plane wave geometry. However, we leave these directions for future investigation.

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The situation in terms of charged black holes is much better. Here we have shown that it is possible to generate a large class of new solutions from the previously known ones. One can use the recent results on the supersymmetric solutions of minimal supergravity in five dimensions [26] to show that it is possible to have solutions which are asymptotically plane wave and yet admit a regular horizon in certain cases. In this context, the solutions generated from M2-branes are perhaps the most interesting, from the point of view of generating asymptotically plane wave black holes. On the other hand, the solutions obtained by deforming the extremal M2, M5 and D3-brane metrics are extremely interesting from the perspective of the holographic dual, for the deformation of the spacetime geometry ought to correspond to some particular deformation of the field theory.

One can illustrate this with the situation of the D3-brane. Consider starting from the near-horizon geometry of the D3-brane, i.e., $AdS_5 \times \mathbf{S}^5$. From the AdS/CFT duality we know that this is related to the $\mathcal{N} = 4$, $d = 4$ Super-Yang-Mills theory. In the dual gauge theory, turning on an irrelevant operator, $\mathrm{Tr}\, F^4$, is believed to correspond to reconnecting the asymptotically flat region in the extremal D3-brane geometry, with the near-horizon $AdS_5 \times \mathbf{S}^5$ region, by recreating the throat geometry that interpolates between the two [27, 28]. (cf. [29]–[32], for related discussions).

We have shown that there is a deformation in the full extremal D3-brane geometry (4.12) (for $p = 3$), which changes the asymptotically flat region $\mathbb{R}^{9,1}$ into $\mathcal{V}_8 \times \mathbb{R}^2$. The resulting geometry is

$$
\begin{aligned}
ds^2 = {} & \left(1 + \frac{R^4}{r^4}\right)^{-1/2} \left[-2\, du\, dv - r^2 \, \left(6 \, \cos^2\theta - 1\right) \, du^2 + \sum_{i=2}^{3} \, \left(dx^i\right)^2\right] + \\
& + \left(1 + \frac{R^4}{r^4}\right)^{1/2} \, \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta \, d\Omega_4^2\right)\right] . \qquad (5.1)
\end{aligned}
$$

The geometry in the near-horizon limit $r \ll 1$, reduces to $AdS_5 \times \mathbf{S}^5$ with a deformation proportional to r^2. In the dual $\mathcal{N} = 4$ gauge theory this would correspond to a deformation by a dimension 6 operator such as $\mathrm{Tr}\, F^3$. One can envisage starting from the $\mathcal{N} = 4$ gauge theory and deforming it first by the dimension eight operator $\mathrm{Tr}\, F^4$, so as to recover the asymptotically flat region and then further deform it by an operator that had dimension six in the AdS limit so as to reproduce the supergravity geometry as in (5.1). One might substantiate the claim further by noting that the full extremal D3-brane geometry is holographically related to non-commutative Super-Yang-Mills theory with a self-dual B-field [33]. One can then ask what is the corresponding deformation in the non-commutative gauge theory which reproduces the geometry (5.1)? This issue deserves to be investigated further. Similar statements can be made for the M2 and M5 brane geometries, but one is handicapped by not having a precise understanding of the dual field theories.

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